SANDRINGHAM SCHOOL

MATHEMATICS DEPARTMENT

Introduction to A Level Maths

SUMMER WORK

A Level Mathematics

Thank you for choosing to study A Level Mathematics in the Sixth Form at Sandringham School.

The Mathematics Department is committed to ensuring that you make good progress throughout your A level course. In order that you make the best possible start to the course, we have prepared a booklet of key topics you need to master before September.

The Task

Work through the questions in this booklet over the summer - you will need to have a good knowledge of these topics before you commence your course in September.

You should have met all the topics before at GCSE. If you are unsure of any topics, go to the Maths Genie website for videos, questions and worked solutions.

Work through the introduction to each chapter, making sure that you understand the examples. Make notes on the key points and mark anything you don't understand.

Then tackle the exercises to ensure you understand the topic thoroughly. The answers are given at the back of the booklet. You must mark your work and correct it where necessary.

We will test you early in the course to check how well you understand these topics, so it is important that you have complete the booklet and fill in the self-assessment page. A practice test is provided at the back of the booklet.

We hope that you will use this introduction to give you a good start to your A level work and that it will help you enjoy and benefit from the course more.

Course Description

Course Title: **Mathematics**
Examination Board: **Edexcel (Pearson)**
Course Code: **9MA0**

Textbooks

You will be given access to the Pearson Edexcel textbooks online at the beginning of the **course.**

Extra support

You may also find the following books useful:

Bridging GCSE & A Level Maths by Mark Rowland
Published by Collins
ISBN-13: 978 0 00820 501 0
Cost: £6.99

AS-Level Maths Head Start
Published by CGP Workbooks
ISBN-13: 978 1 78294 792 9
Cost: £5.95

The excellent website www.examsolutions.co.uk has tutorial clips for the entire course, as well as a selection of GCSE topics

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As you work through this booklet you should make a note on this checklist of where you needed help. If you are still unsure about a topic, tick the final column.

Please do not just pretend you are ok with these topics if you are struggling! We are here to help! We will put on extra sessions to help you sort out these problems early on in the course.

EXERCISE CHECK LIST

TOPIC	Exercise	I was fine on this exercise	I got help on this exercise and now it's ok	I still have a problem with this topic
Removing brackets	A B (DOTs)			
Linear equations	A B (brackets) C (fractions)			
Simultaneous equations	A			
Factorising	A B (quadratics)			
Change the subject of the formula	A B C			
Solving quadratic equations	A			
Indices	A B (fractional and negative)			
Surds	A Simplifying B Calculating C Rationalising			
Completing the Square	A			
Practice Test				

Chapter 1: REMOVING BRACKETS

To remove a single bracket, we multiply every term in the bracket by the number or the expression on the outside:

Examples

1) $3(x + 2y)$ $= 3x + 6y$

2) $-2(2x - 3)$ $= (-2)(2x) + (-2)(-3)$
$= -4x + 6$

To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including

* the smiley face method
* FOIL (Fronts Outers Inners Lasts)
* using a grid.

Examples:

1) $(x + 1)(x + 2)$ $= x(x + 2) + 1(x + 2)$

or

$(x + 1)(x + 2)$ $= x^2 + 2 + 2x + x$
$= x^2 + 3x + 2$

or

	x	1
x	x^2	x
2	$2x$	2

$(x + 1)(x + 2)$ $= x^2 + 2x + x + 2$
$= x^2 + 3x + 2$

2) $(x - 2)(2x + 3)$ $= x(2x + 3) - 2(2x + 3)$
$= 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

or

$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$

or

	x	-2
$2x$	$2x^2$	$-4x$
3	$3x$	-6

$(2x + 3)(x - 2)$ $= 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

EXERCISE A Multiply out the following brackets and simplify.

1. $7(4x + 5)$
2. $-3(5x - 7)$
3. $5a – 4(3a - 1)$
4. $4y + y(2 + 3y)$
5. $-3x – (x + 4)$
6. $5(2x - 1) – (3x - 4)$
7. $(x + 2)(x + 3)$
8. $(t - 5)(t - 2)$
9. $(2x + 3y)(3x – 4y)$
10. $4(x - 2)(x + 3)$
11. $(2y - 1)(2y + 1)$
12. $(3 + 5x)(4 – x)$

Two Special Cases

Perfect Square:	**Difference of two squares:**
$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$	$(x - a)(x + a) = x^2 – a^2$
$(2x - 3)^2 = (2x – 3)(2x – 3) = 4x^2 – 12x + 9$	$(x - 3)(x + 3) = x^2 – 3^2$
	$= x^2 – 9$

EXERCISE B Multiply out

1. $(x - 1)^2$
2. $(3x + 5)^2$
3. $(7x - 2)^2$
4. $(x + 2)(x - 2)$
5. $(3x + 1)(3x - 1)$
6. $(5y - 3)(5y + 3)$

Chapter 2: LINEAR EQUATIONS

When solving an equation, you must remember that whatever you do to one side must also be done to the other. You are therefore allowed to

- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in x. A linear equation does not contain any x^2 or x^3 terms.

More help on solving equations can be obtained by downloading the leaflet available at this website:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simplelinear.pdf

Example 1: Solve the equation $64 - 3x = 25$

Solution: There are various ways to solve this equation. One approach is as follows:

Step 1: Add $3x$ to both sides (so that the x term is positive): $64 = 3x + 25$

Step 2: Subtract 25 from both sides: $39 = 3x$

Step 3: Divide both sides by 3: $13 = x$

So the solution is $x = 13$.

Example 2: Solve the equation $6x + 7 = 5 - 2x$.

Solution:
Step 1: Begin by adding $2x$ to both sides (to ensure that the x terms are together on the same side): $8x + 7 = 5$

Step 2: Subtract 7 from each side: $8x = -2$

Step 3: Divide each side by 8: $x = -¼$

Exercise A: Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$

2) $5x - 2 = 13$

3) $11 - 4x = 5$

4) $5 - 7x = -9$

5) $11 + 3x = 8 - 2x$

6) $7x + 2 = 4x - 5$

Example 3: Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: (taking care of the negative signs) $6x - 4 = 20 - 3x - 6$

Step 2: Simplify the right hand side: $6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $9x - 4 = 14$

Step 4: Add 4: $9x = 18$

Step 5: Divide by 9: $x = 2$

Exercise B: Solve the following equations.

1) $5(2x - 4) = 4$

2) $4(2 - x) = 3(x - 9)$

3) $8 - (x + 3) = 4$

4) $14 - 3(2x + 3) = 2$

EQUATIONS CONTAINING FRACTIONS

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

Example 4: Solve the equation $\frac{y}{2} + 5 = 11$

Solution:
Step 1: Multiply through by 2 (the denominator in the fraction): $y + 10 = 22$

Step 2: Subtract 10: $y = 12$

Example 5: Solve the equation $\frac{1}{3}(2x + 1) = 5$

Solution:
Step 1: Multiply by 3 (to remove the fraction) $2x + 1 = 15$

Step 2: Subtract 1 from each side $2x = 14$

Step 3: Divide by 2 $x = 7$

When an equation contains two fractions, you need to multiply by the lowest common denominator. This will then remove both fractions.

Example 6: Solve the equation $\frac{x+1}{4}+\frac{x+2}{5}=2$

Solution:

Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator $\frac{20(x+1)}{4}+\frac{20(x+2)}{5}=40$

Step 3: Simplify the left hand side: $\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}}+\frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}}=40$

$5(x + 1) + 4(x + 2) = 40$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13 $9x = 27$

Step 7: Divide by 9: $x = 3$

Example 7: Solve the equation $x+\frac{x-2}{4}=2-\frac{3-5x}{6}$

Solution: The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x+\frac{12(x-2)}{4}=24-\frac{12(3-5x)}{6}$$

Simplify $12x+3(x-2)=24-2(3-5x)$

Expand brackets $12x+3x-6=24-6+10x$

Simplify $15x-6=18+10x$

Subtract $10x$ $5x-6=18$

Add 6 $5x = 24$

Divide by 5 $x = 4.8$

Exercise C: Solve these equations

1) $\frac{1}{2}(x+3)=5$

2) $\frac{2x}{3}-1=\frac{x}{3}+4$

Exercise C (continued)

3) $\frac{y}{4}+3=5-\frac{y}{3}$

4) $\frac{x-2}{7}=2+\frac{3-x}{14}$

5) $\frac{7x-1}{2}=13-x$

6) $\frac{y-1}{2}+\frac{y+1}{3}=\frac{2y+5}{6}$

7) $2x+\frac{x-1}{2}=\frac{5x+3}{3}$

8) $2-\frac{5}{x}=\frac{10}{x}-1$

Chapter 3: SIMULTANEOUS EQUATIONS

An example of a pair of simultaneous equations is $3x + 2y = 8$ ①
$5x + y = 11$ ②

In these equations, x and y stand for two numbers. We can solve these equations in order to find the values of x and y by eliminating one of the letters from the equations.

In these equations it is simplest to eliminate y. We do this by making the coefficients of y the same in both equations. This can be achieved by multiplying equation ② by 2, so that both equations contain $2y$:

$3x + 2y = 8$ ①
$10x + 2y = 22$ $2\times② = ③$

To eliminate the y terms, we subtract equation ③ from equation ①. We get: $7x = 14$
i.e. $x = 2$

To find y, we substitute $x = 2$ into one of the original equations. For example if we put it into ②:

$10 + y = 11$
$y = 1$

Therefore the solution is $x = 2$, $y = 1$.

Remember: You can check your solutions by substituting both x and y into the original equations.

Example: Solve $2x + 5y = 16$ ①
$3x - 4y = 1$ ②

Solution: We begin by getting the same number of x or y appearing in both equation. We can get $20y$ in both equations if we multiply the top equation by 4 and the bottom equation by 5:

$8x + 20y = 64$ ③
$15x - 20y = 5$ ④

As the SIGNS in front of $20y$ are DIFFERENT, we can eliminate the y terms from the equations by ADDING:

$23x = 69$ ③+④
i.e. $x = 3$

Substituting this into equation ① gives:

$6 + 5y = 16$
$5y = 10$
So… $y = 2$
The solution is $x = 3$, $y = 2$.

If you need **more help** on solving simultaneous equations, you can download a booklet from the following website:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simultaneous1.pdf

Exercise A:

Solve the pairs of simultaneous equations in the following questions:

1) $x + 2y = 7$
$3x + 2y = 9$

2) $x + 3y = 0$
$3x + 2y = -7$

3) $3x - 2y = 4$
$2x + 3y = -6$

4) $9x - 2y = 25$
$4x - 5y = 7$

5) $4a + 3b = 22$
$5a - 4b = 43$

6) $3p + 3q = 15$
$2p + 5q = 14$

Chapter 4: FACTORISING

Common factors

We can factorise some expressions by taking out a common factor.

Example 1: Factorise $12x - 30$

Solution: 6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket:

$$12x - 30 = 6(2x - 5)$$

Example 2: Factorise $6x^2 - 2xy$

Solution: 2 is a common factor to both 6 and 2. Both terms also contain an x.
So we factorise by taking $2x$ outside a bracket.

$$6x^2 - 2xy = 2x(3x - y)$$

Example 3: Factorise $9x^3y^2 - 18x^2y$

Solution: 9 is a common factor to both 9 and 18.
The highest power of x that is present in both expressions is x^2.
There is also a y present in both parts.
So we factorise by taking $9x^2y$ outside a bracket:

$$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$$

Example 4: Factorise $3x(2x - 1) - 4(2x - 1)$

Solution: There is a common bracket as a factor.
So we factorise by taking $(2x - 1)$ out as a factor.
The expression factorises to $(2x - 1)(3x - 4)$

Exercise A

Factorise each of the following

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

Factorising quadratics

Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$

The method is:

Step 1: Form two brackets $(x \ \ldots \ )(x \ \ldots \ )$

Step 2: Find two numbers that multiply to give c and add to make b. These two numbers get written at the other end of the brackets.

Example 1: Factorise $x^2 - 9x - 10$.

Solution: We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.

Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$

The method is:

Step 1: Find two numbers that multiply together to make ac and add to make b.

Step 2: Split up the bx term using the numbers found in step 1.

Step 3: Factorise the front and back pair of expressions as fully as possible.

Step 4: There should be a common bracket. Take this out as a common factor.

Example 2: Factorise $6x^2 + x - 12$.

Solution: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = 6x^2 \underbrace{- 8x} \underbrace{+ 9x} - 12$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x - a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

$16x^2 - 25 = (2x)^2 - 5^2 = (2x + 5)(2x - 5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x + 4)(x - 4)$

and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x + 4y)(x - 4y)$

Factorising by pairing

We can factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) - 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)

$= (2x + y)(x - 1)$

If you need **more help** with factorising, you can download a booklet from this website:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-factorisingquadratics.pdf

Exercise B

Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$ (factorise by taking out a common factor)

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2y$

14) $8(x+1)^2 - 2(x+1) - 10$

Chapter 5: CHANGING THE SUBJECT OF A FORMULA

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation – we must do the same to both sides in order to keep the equation balanced.

Example 1: Make x the subject of the formula $y = 4x + 3$.

Solution:

	$y = 4x + 3$
Subtract 3 from both sides:	$y - 3 = 4x$
Divide both sides by 4;	$\frac{y-3}{4} = x$

So $x = \frac{y-3}{4}$ is the same equation but with x the subject.

Example 2: Make x the subject of $y = 2 - 5x$

Solution: Notice that in this formula the x term is negative.

	$y = 2 - 5x$	
Add $5x$ to both sides	$y + 5x = 2$	(the x term is now positive)
Subtract y from both sides	$5x = 2 - y$	
Divide both sides by 5	$x = \frac{2-y}{5}$	

Example 3: The formula $C = \frac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.

We can rearrange to make F the subject.

	$C = \frac{5(F-32)}{9}$	
Multiply by 9	$9C = 5(F-32)$	(this removes the fraction)
Expand the brackets	$9C = 5F - 160$	
Add 160 to both sides	$9C + 160 = 5F$	
Divide both sides by 5	$\frac{9C+160}{5} = F$	

Therefore the required rearrangement is $F = \frac{9C+160}{5}$.

Exercise A

Make x the subject of each of these formulae:

1) $y = 7x - 1$

2) $y = \frac{x+5}{4}$

3) $4y = \frac{x}{3} - 2$

4) $y = \frac{4(3x-5)}{9}$

Rearranging equations involving squares and square roots

Example 4: Make x the subject of $x^2 + y^2 = w^2$

Solution: $x^2 + y^2 = w^2$

Subtract y^2 from both sides: $x^2 = w^2 - y^2$ (this isolates the term involving x)

Square root both sides: $x = \pm\sqrt{w^2 - y^2}$

Remember that you can have a positive or a negative square root. We cannot simplify the answer any more.

Example 5: Make a the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Solution: $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Multiply by 4 $4t = \sqrt{\frac{5a}{h}}$

Square both sides $16t^2 = \frac{5a}{h}$

Multiply by h: $16t^2h = 5a$

Divide by 5: $\frac{16t^2h}{5} = a$

Exercise B:

Make t the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

More difficult examples

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In these questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

Example 6: Make *t* the subject of the formula $a - xt = b + yt$

Solution: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add *xt* to both sides: $a = b + yt + xt$

Now put the terms without a *t* on the left hand side:

Subtract *b* from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by $(y + x)$: $\frac{a-b}{y+x} = t$

So the required equation is $t = \frac{a-b}{y+x}$

Example 7: Make *W* the subject of the formula $T - W = \frac{Wa}{2b}$

Solution: This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by $2b$: $2bT - 2bW = Wa$

Add $2bW$ to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by $a + 2b$: $W = \frac{2bT}{a+2b}$

If you need more help you can download an information booklet on rearranging equations from the following website:

http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-formulae2-tom.pdf

Exercise C

Make *x* the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \frac{2x+3}{5x-2}$

4) $\frac{x}{a} = 1 + \frac{x}{b}$

Chapter 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:
* factorising
* the quadratic formula

Note that not all quadratic equations can be solved by factorising. The quadratic formula can always be used however.

Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example 1 : Solve $x^2 - 3x + 2 = 0$

Factorise $(x-1)(x-2) = 0$
Either $(x-1) = 0$ or $(x-2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Example 2: Solve $x^2 - 2x = 0$

Factorise: $x(x-2) = 0$
Either $x = 0$ or $(x-2) = 0$
So $x = 0$ or $x = 2$

Method 2: Using the formula

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3: Solve the equation $2x^2 - 5 = 7 - 3x$

Solution: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$
We can then tell that $a = 2$, $b = 3$ and $c = -12$.
Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)

If we have a calculator, we can evaluate these roots to get: $x = 1.81$ or $x = -3.31$

If you need more help with the work in this chapter, there is an information booklet downloadable from this web site:

http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-quadraticequations.pdf

EXERCISE A

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$ b) $x^2 - 3x - 4 = 0$

c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$ b) $x^2 - 4x = 0$

c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$ b) $8x^2 - 24x + 10 = 0$

4) Use the formula to solve the following equations to 3 significant figures. Some of the equations can't be solved.

a) $x^2 + 7x + 9 = 0$ b) $6 + 3x = 8x^2$

c) $4x^2 - x - 7 = 0$ d) $x^2 - 3x + 18 = 0$

e) $3x^2 + 4x + 4 = 0$ f) $3x^2 = 13x - 16$

Chapter 7: INDICES

Basic rules of indices

y^4 means $y \times y \times y \times y$. 4 is called the **index** (plural: indices), **power** or **exponent** of *y*.

There are 3 basic rules of indices:

1) $a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$
2) $a^m \div a^n = a^{m-n}$ e.g. $3^8 \times 3^6 = 3^2$
3) $(a^m)^n = a^{mn}$ e.g. $\left(3^2\right)^5 = 3^{10}$

Further examples

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the numbers and multiply the *a*'s)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the numbers and multiply the *c*'s)

$24d^7 \div 3d^2 = \frac{24d^7}{3d^2} = 8d^5$ (divide the numbers and divide the *d* terms i.e. by subtracting the powers)

Exercise A

Simplify the following:

1) $b \times 5b^5 =$ (Remember that $b = b^1$)

2) $3c^2 \times 2c^5 =$

3) $b^2c \times bc^3 =$

4) $2n^6 \times (-6n^2) =$

5) $8n^8 \div 2n^3 =$

6) $d^{11} \div d^9 =$

7) $\left(a^3\right)^2 =$

8) $\left(-d^4\right)^3 =$

More complex powers

Zero index:
Recall from GCSE that

$$a^0 = 1.$$

This result is true for any non-zero number *a*.

Therefore $5^0 = 1$ $\left(\frac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

Negative powers

A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \frac{1}{a}$

Therefore $5^{-1} = \frac{1}{5}$

$0.25^{-1} = \frac{1}{0.25} = 4$

$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$ (you find the reciprocal of a fraction by swapping the top and bottom over)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$

Fractional powers:

Fractional powers correspond to roots: $a^{1/2} = \sqrt{a}$ $a^{1/3} = \sqrt[3]{a}$ $a^{1/4} = \sqrt[4]{a}$

In general:

$$a^{1/n} = \sqrt[n]{a}$$

Therefore:

$8^{1/3} = \sqrt[3]{8} = 2$ $25^{1/2} = \sqrt{25} = 5$ $10000^{1/4} = \sqrt[4]{10000} = 10$

A more general fractional power can be dealt with in the following way: $a^{m/n} = \left(a^{1/n}\right)^m$

So $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

$$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \left(\sqrt{\frac{36}{25}}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$$

Exercise B:

Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(1/9\right)^{1/2}$

4) 5^{-2}

5) 18^0

6) 7^{-1}

7) $27^{2/3}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-2/3}$

10) $(0.04)^{1/2}$

11) $\left(\frac{8}{27}\right)^{2/3}$

12) $\left(\frac{1}{16}\right)^{-3/2}$

Simplify each of the following:

13) $2a^{1/2} \times 3a^{5/2}$

14) $x^3 \times x^{-2}$

15) $\left(x^2 y^4\right)^{1/2}$

Chapter 8: Surds

A surd is an irrational number expressed as a root.

Rules

- $\sqrt{ab} = \sqrt{a}\sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Simplifying

To simplify a surd, we extract any rational numbers from it. We do this by removing all square factors.

$\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

Exercise A

Simplify

a $\sqrt{12}$ ***b*** $\sqrt{28}$ ***c*** $\sqrt{80}$

d $\sqrt{27}$ ***e*** $\sqrt{24}$ ***f*** $\sqrt{128}$

Adding

To add surds, we apply the same rules as algebra. Sometimes we need to simplify surds first.
$\sqrt{3} + \sqrt{3} = 2\sqrt{3}$
$\sqrt{2} + \sqrt{8} = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

Multiplying

To multiply surds, using the following tip will help: Multiply the non-surd first, and the surd second. Multiplying brackets follows the same procedures as algebra.

$2\sqrt{3} \times 2\sqrt{5} = 4\sqrt{15}$

$(2 + \sqrt{3}) \times (3 - \sqrt{3}) = 6 + 3\sqrt{3} - 2\sqrt{3} - 3 = 3 + \sqrt{3}$

Exercise B

1. Express in the form $a + b\sqrt{3}$

a $\sqrt{3}(2 + \sqrt{3})$ ***b*** $4 - \sqrt{3} - 2(1 - \sqrt{3})$ ***c*** $(1 + \sqrt{3})(2 + \sqrt{3})$

2. Simplify

a $(\sqrt{5} + 1)(2\sqrt{5} + 3)$ ***b*** $(1 - \sqrt{2})(4\sqrt{2} - 3)$ ***c*** $(2\sqrt{7} + 3)^2$

Rationalising the denominator

This means writing as an equivalent fraction without any surds in the denominator. To rationalise, we multiply by "1" to remove any surds from the denominator. For a single term we multiply by this term. Where there is more than one term, we use the same terms with the opposite sign in between.

To rationalise $\frac{3}{\sqrt{7}}$

We write $\frac{3}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

To rationalise $\frac{3+\sqrt{5}}{2-\sqrt{7}}$

We write

$\frac{(3+\sqrt{2})}{(5-\sqrt{2})} \times \frac{(5+\sqrt{2})}{(5+\sqrt{2})}$

$= \frac{15+5\sqrt{2}+3\sqrt{2}+2}{25-5\sqrt{2}+5\sqrt{2}-2} = \frac{17+8\sqrt{2}}{23}$

Exercise C

For each of the below fractions, rationalise the denominator

a. $\frac{2}{\sqrt{3}}$

b. $\frac{1}{\sqrt{2}+1}$

c. $\frac{6}{\sqrt{5}-\sqrt{3}}$

d. $\frac{1+\sqrt{3}}{2+\sqrt{3}}$

e. $\frac{\sqrt{3}-5}{2\sqrt{3}-4}$

Chapter 9: COMPLETING THE SQUARE

There is a formula for C.T.S which you can use – but it is better to learn and understand the method rather than substitute into a formula:

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

Completing the square is used to write out a quadratic equation:
$x^2 + 2bx + b^2 = (x + b)^2$
$x^2 - 2bx + b^2 = (x - b)^2$

To complete the square of the function $x^2 + 2bx$ you need a further term b^2.

So the completed square form is

$$x^2 + 2bx = (x + b)^2 - b^2$$

Similarly
$x^2 - 2bx = (x - b)^2 - b^2$

Example 1:
Complete the square for the expression $x^2 + 8x$
$x^2 + 8x$
$=(x + 4)^2 - 4^2$
$=(x + 4)^2 - 16$

Example 2:
Complete the square for expressions

a) $x^2 + 12x$

$= (x - 6)^2 - 6^2$

$= (x - 6)^2 - 36$

b) $2x^2 - 10x$

$= 2(x^2 - 5x)$

$= 2\left[\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right]$

$= 2\left(x - \frac{5}{2}\right)^2 - \frac{25}{2}$

Exercise A:

Complete the square for the expressions:

1. $x^2 + 4x$
2. $x^2 - 6x$
3. $x^2 - 16x$
4. $x^2 + x$
5. $x^2 - 14x$
6. $2x^2 + 16x$
7. $3x^2 - 24x$
8. $2x^2 - 4x$
9. $5x^2 + 20x$
10. $2x^2 - 5x$
11. $3x^2 + 9x$
12. $3x^2 - x$

Practice Booklet Test

Sit this in test conditions to practise what you have learned and revised.

You may NOT use a calculator

1. Expand and simplify
   (a) $(2x + 3)(2x – 1)$ (b) $(a + 3)^2$ (c) $4x(3x – 2) – x(2x + 5)$

2. Factorise
   (a) $x^2 – 7x$ (b) $y^2 – 64$ (c) $2x^2 + 5x – 3$ (d) $6t^2 – 13t + 5$

3. Simplify
   (a) $\frac{4x^3y}{8x^2y^3}$ (b) $\frac{3x+2}{3} + \frac{4x-1}{6}$

4. Solve the following equations
   (a) $\frac{h-1}{4} + \frac{3h}{5} = 4$ (b) $x^2 – 8x = 0$ (c) $p^2 + 4p = 12$

5. Write each of the following as single powers of x and / y
   (a) $\frac{1}{x^4}$ (b) $(x^2y)^3$ (c) $\frac{x^5}{x^{-2}}$

6. Work out the values of the following, giving your answers as fractions
   (a) 4^{-2} (b) 10^0 (c) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

7. Solve the simultaneous equations
   $3x – 5y = -11$
   $5x – 2y = 7$

8. Rearrange the following equations to make x the subject
   (a) $v^2 = u^2 + 2ax$ (b) $V = \frac{1}{3}\pi x^2 h$ (c) $y = \frac{x+2}{x+1}$

9. Solve $5x^2 – x – 1 = 0$ giving your solutions in surd form

10. Express $\sqrt{200}$ in the form $k\sqrt{2}$

11. Rationalise the denominator $\frac{1-\sqrt{2}}{3-\sqrt{8}}$

12. If $x^2 + 6x + 4 = (x + p)^2 + q$
    Find the values of p and q

SOLUTIONS TO THE EXERCISES

CHAPTER 1:

Ex A

1) $28x + 35$ 2) $-15x + 21$ 3) $-7a + 4$ 4) $6y + 3y^2$ 5) $-4x - 4$
6) $7x - 1$ 7) $x^2 + 5x + 6$ 8) $t^2 - 7t + 10$ 9) $6x^2 + xy - 12y^2$
10) $4x^2 + 4x - 24$ 11) $4y^2 - 1$ 12) $12 + 17x - 5x^2$

Ex B

1) $x^2 - 2x + 1$ 2) $9x^2 + 30x + 25$ 3) $49x^2 - 28x + 4$ 4) $x^2 - 4$
5) $9x^2 - 1$ 6) $25y^2 - 9$

CHAPTER 2

Ex A

1) 7 2) 3 3) 1½ 4) 2 5) -3/5 6) -7/3

Ex B

1) 2.4 2) 5 3) 1 4) ½

Ex C

1) 7 2) 15 3) 24/7 4) 35/3 5) 3 6) 2 7) 9/5 8) 5

Ex D

1) 34, 36, 38 2) 9.875, 29.625 3) 24, 48

CHAPTER 3

1) $x = 1, y = 3$ 2) $x = -3, y = 1$ 3) $x = 0, y = -2$ 4) $x = 3, y = 1$
5) $a = 7, b = -26$) $p = 11/3, q = 4/3$

CHAPTER 4

Ex A

1) $x(3 + y)$ 2) $2x(2x - y)$ 3) $pq(q - p)$ 4) $3q(p - 3q)$ 5) $2x^2(x - 3)$ 6) $4a^3b^2(2a^2 - 3b^2)$
7) $(y - 1)(5y + 3)$

Ex B

1) $(x - 3)(x + 2)$ 2) $(x + 8)(x - 2)$ 3) $(2x + 1)(x + 2)$ 4) $x(2x - 3)$ 5) $(3x - 1)(x + 2)$
6) $(2y + 3)(y + 7)$ 7) $(7y - 3)(y - 1)$ 8) $5(2x - 3)(x + 2)$ 9) $(2x + 5)(2x - 5)$ 10) $(x - 3)(x - y)$
11) $4(x - 2)(x - 1)$ 12) $(4m - 9n)(4m + 9n)$ 13) $y(2y - 3a)(2y + 3a)$ 14) $2(4x - 1)(x + 2)$

CHAPTER 5

Ex A

1) $x = \dfrac{y+1}{7}$ 2) $x = 4y - 5$ 3) $x = 3(4y + 2)$ 4) $x = \dfrac{9y+20}{12}$

Ex B

1) $t = \dfrac{32rP}{w}$ 2) $t = \pm\sqrt{\dfrac{32rP}{w}}$ 3) $t = \pm\sqrt{\dfrac{3V}{\pi h}}$ 4) $t = \dfrac{P^2 g}{2}$ 5) $t = v - \dfrac{Pag}{w}$ 6) $t = \pm\sqrt{\dfrac{r-a}{b}}$

Ex C

1) $x = \dfrac{c-3}{a-b}$ 2) $x = \dfrac{3a+2k}{k-3}$ 3) $x = \dfrac{2y+3}{5y-2}$ 4) $x = \dfrac{ab}{b-a}$

CHAPTER 6

1) a) -1, -2 b) -1, 4 c) -5, 3 2) a) 0, -3 b) 0, 4 c) 2, -2
3) a) -1/2, 4/3 b) 0.5, 2.5 4) a) -5.30, -1.70 b) 1.07, -0.699 c) -1.20, 1.45
d) no solutions e) no solutions f) no solutions

CHAPTER 7

Ex A

1) $5b^6$ 2) $6c^7$ 3) b^3c^4 4) $-12n^8$ 5) $4n^5$ 6) d^2 7) a^6 8) $-d^{12}$

Ex B

1) 2 2) 3 3) 1/3 4) 1/25 5) 1 6) 1/7 7) 9 8) 9/4 9) ¼ 10) 0.2 11) 4/9 12) 64
13) $6a^3$ 14) x 15) xy^2

CHAPTER 8

Ex A

$\boldsymbol{a}\ \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$ $\boldsymbol{b}\ = \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$ $\boldsymbol{c}\ = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

$\boldsymbol{d}\ = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$ $\boldsymbol{e}\ = \sqrt{4} \times \sqrt{6} = 2\sqrt{6}$ $\boldsymbol{f}\ = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}$

Ex B

$\boldsymbol{a}\ = 3 + 2\sqrt{3}$ $\boldsymbol{b}\ = 4 - \sqrt{3} - 2 + 2\sqrt{3} = 2 + \sqrt{3}$ $\boldsymbol{c}\ = 2 + \sqrt{3} + 2\sqrt{3} + 3 = 5 + 3\sqrt{3}$

$\boldsymbol{d}\ = 10 + 3\sqrt{5} + 2\sqrt{5} + 3$
$= 13 + 5\sqrt{5}$

$\boldsymbol{e}\ = 4\sqrt{2} - 3 - 8 + 3\sqrt{2}$
$= 7\sqrt{2} - 11$

$\boldsymbol{f}\ = 28 + 12\sqrt{7} + 9$
$= 37 + 12\sqrt{7}$

Ex C

a. $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

b. $\frac{1}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{\sqrt{2}-1}{2-\sqrt{2}+\sqrt{2}-1} = \sqrt{2} - 1$

c. $\frac{6}{(\sqrt{5}-\sqrt{3})} \times \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} = \frac{6(\sqrt{5}+\sqrt{3})}{5-\sqrt{15}+\sqrt{15}-3} = \frac{6(\sqrt{5}+\sqrt{3})}{2} = 3\sqrt{5} + 3\sqrt{3}$

d. $\frac{(1+\sqrt{3})}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{2+2\sqrt{3}-\sqrt{3}-3}{4-2\sqrt{3}+2\sqrt{3}-3} = \frac{\sqrt{3}-1}{1} = \sqrt{3} - 1$

e. $\frac{\sqrt{3}-5}{2\sqrt{3}-4} \times \frac{(2\sqrt{3}+4)}{(2\sqrt{3}+4)} = \frac{6+4\sqrt{3}-10\sqrt{3}-20}{12-8\sqrt{3}+8\sqrt{3}-16} = \frac{3\sqrt{3}+7}{2}$

CHAPTER 9

Ex A

1. $(x + 2)^2 - 4$
2. $(x - 3)^2 - 9$
3. $(x - 8)^2 - 64$
4. $\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$
5. $(x - 7)^2 - 49$
6. $2(x + 4)^2 - 32$
7. $3(x - 4)^2 - 48$
8. $2(x - 1)^2 - 2$
9. $5(x + 2)^2 - 20$
10. $2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8}$
11. $3\left(x + \frac{3}{2}\right)^2 - \frac{27}{4}$
12. $3\left(x - \frac{1}{2}\right)^2 - \frac{1}{12}$

SOLUTIONS TO PRACTICE BOOKLET TEST

1) a) $4x^2 + 4x - 3$ b) $a^2 + 6a + 9$ c) $10x^2 - 13x$

2) a) $x(x - 7)$ b) $(y + 8)(y - 8)$ c) $(2x - 1)(x + 3)$ d) $(3t - 5)(2t - 1)$

3) a) $\frac{x}{2y^2}$ b) $\frac{10x+3}{6}$

4) a) h = 5 b) $x = 0$ or $x = 8$ c) p = -6 or p = 2

5) a) x^{-4} b) x^6y^3 c) x^7

6) a) $\frac{1}{16}$ b) 1 c) $\frac{2}{3}$

7) $x = 3, y = 4$

8) a) $x = \frac{v^2 - u^2}{2a}$ b) $x = \sqrt{\frac{3V}{\pi h}}$ c) $x = \frac{2-y}{y-1}$

9) $x = \frac{1 \pm \sqrt{21}}{10}$

10) $\sqrt{200} = \sqrt{100}\sqrt{2} = 10\sqrt{2}$

11) $\frac{(1-\sqrt{2})}{(3-2\sqrt{2})} \times \frac{(3+2\sqrt{2})}{(3+2\sqrt{2})} = \frac{3+2\sqrt{2}-3\sqrt{2}-4}{9-6\sqrt{2}+6\sqrt{2}-8} = \frac{-1-\sqrt{2}}{1} = -1 - \sqrt{2}$

12) $p = 3, q = -5$